Problem 1.1

A harmonic motion has an amplitude of 0.20 cm and a period of 0.15 s. Determine the maximum velocity and acceleration.

Solution

An object in harmonic motion has a position given by

$$x(t) = A \sin \frac{2\pi}{\tau} t,$$

where A is the amplitude and τ is the period. Change the given amplitude to meters ($0.20 \text{ cm} = 0.0020 \text{ m}$) and plug in the numbers.

$$\begin{aligned} x(t) &= 0.0020 \sin \frac{2\pi}{0.15} t \\ &= \frac{1}{500} \sin \frac{40\pi}{3} t \end{aligned}$$

Differentiate the position with respect to time to get the velocity.

$$\begin{aligned} \dot{x}(t) &= \frac{1}{500} \cdot \frac{40\pi}{3} \cos \frac{40\pi}{3} t \\ &= \frac{2\pi}{75} \cos \frac{40\pi}{3} t \end{aligned}$$

We conclude that the maximum velocity is

$$v_{\max} = \frac{2\pi}{75} \approx 0.0838 \ \frac{\text{m}}{\text{s}}.$$

Differentiate the velocity with respect to time to get the acceleration.

$$\begin{aligned} \ddot{x}(t) &= -\frac{2\pi}{75} \cdot \frac{40\pi}{3} \sin \frac{40\pi}{3} t \\ &= -\frac{16\pi^2}{45} \sin \frac{40\pi}{3} t \end{aligned}$$

We conclude that the maximum acceleration is

$$a_{\max} = \frac{16\pi^2}{45} \approx 3.51 \ \frac{\text{m}}{\text{s}}.$$